LAST Name: _____

First Name: _____

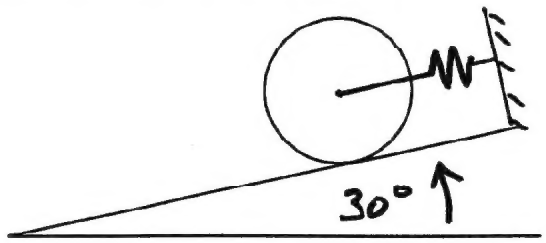
Student Number: _____

MIE 200F - Quiz number 6a - November 14, 2001
quiz duration = 25 minutes

The uniform disk rolls from rest down the hill without slipping. Mass of disk is 30 kg. Radius of disk is 0.5 meters. The spring is unstretched when the disk starts to roll. The spring constant is 20 N/m.

(a) Find the velocity of the center of gravity of the disk, after it has rolled 12 meters.

(b) For a rolling uniform disk, what is the ratio of the translational to the rotational kinetic energies?



$g = 9.81\ m/s^2 \downarrow$ $\quad T_2 = T_1 + U_{1\to 2}$

$I_{c\ of\ g}$ for a rod $= m L^2/12$ $\quad I_o = I_{c\ of\ g} + m d^2$

$I_{c\ of\ g}$ for a uniform disk $= m R^2/2$

$\vec{V}_P = \vec{V}_Q + \vec{V}_{P/Q}$ $\quad |\vec{V}_{P/Q}| = \omega R_{PQ}$

$\vec{a}_{P/Q} = -(\omega^2 R_{PQ})\hat{e}_r + (\alpha r)\hat{e}_\theta$ $\quad \vec{a}_P = \vec{a}_Q + \vec{a}_{P/Q}$

$T = \frac{1}{2} m (v_{c\ of\ g})^2 + \frac{1}{2} I_{c\ of\ g} \omega^2 = \frac{1}{2} I_o \omega^2$

(a) $T_2 = \cancel{T_1}^{0} + U_{1-2}$

$\frac{1}{2} I_o \omega^2 = -mgdh - \frac{1}{2} k (x_2^2 - \cancel{x_1^2}_{0})$

$I_o = m\frac{R^2}{2} + mR^2 = 3mR^2/2$

$(\frac{1}{2})(3mR^2/2)\,\omega^2 = -(30)(9.81)(-12 \sin 30) - \frac{20}{2}(12)^2$

$(\frac{1}{2})(3)(30)(\frac{0.5^2}{2})\,\omega^2 = 1766 - 1440 = 326$

$\omega^2 = 58 \Rightarrow \omega = 7.6\ s^{-1}$

$\vec{V}_{c\ of\ g} = \omega r = (7.6)(.5) = 3.81\ m/s$ 30° downhill

(b) $\cancel{\frac{1}{2}} m (V_{c\ of\ g})^2 : \cancel{\frac{1}{2}} I_{c\ of\ g} \omega^2$

$m (\omega R)^2 : \frac{mR^2}{2} \omega^2$

$2 : 1$

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MIE 200F - Quiz number 6a - November 15/99
quiz duration = 25 minutes

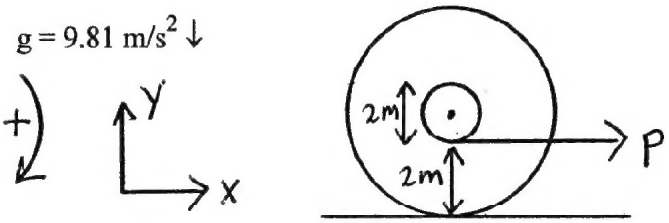
A non-uniform wheel has a radius of gyration about its center of gravity equal to 2 meters. Its radius is 3 meters, and its mass is 20 kg. It is rolling to the left, with an angular velocity $\omega = -5\ s^{-1}$, and an angular acceleration $\alpha = +2\ s^{-2}$. The force P is applied to a rope wound around its inner hub as shown. The wheel does not slip on the road. Find the magnitude and direction of the frictional force F acting on the wheel.

$\Sigma \vec{F} = m \vec{a}_{c\ of\ g}$ $\quad \Sigma \vec{M}_o = I_o \vec{\alpha}$ $\quad \Sigma \vec{M}_{c\ of\ g} = I_{c\ of\ g} \vec{\alpha}$

$\vec{V}_P = \vec{V}_Q + \vec{V}_{P/Q}$ $\quad |\vec{V}_{P/Q}| = \omega R_{PQ}$

$\vec{a}_P = \vec{a}_Q + \vec{a}_{P/Q}$ $\quad \vec{a}_{P/Q} = -(\omega^2 R_{PQ})\hat{e}_r + (\alpha r)\hat{e}_\theta$

$g = 9.81\ m/s^2 \downarrow$



$I_{c\ of\ g} = mk^2 = (20kg)(2m)^2 = 80kg \cdot m^2$

α is positive $\Rightarrow \Sigma \vec{M}$ must be positive $\Rightarrow$ friction is to the left.

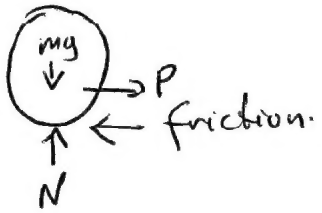
$\Sigma \vec{F} = m \vec{a}_{c\ of\ g}$

$\Rightarrow P - friction = 20 a_x = (20)(\alpha R)$ ③

① $\quad P - friction = 60\alpha = 120$

Also $\quad \Sigma \vec{M}_{c\ of\ g} = I_{c\ of\ g}\alpha$

② $\Rightarrow \quad -P + 3\,Friction = 80\alpha = 160$



Combine ① & ② $\Rightarrow$ $\quad Friction = 140$

Friction = $-140\,\hat{i}$ Newtons.

OR //

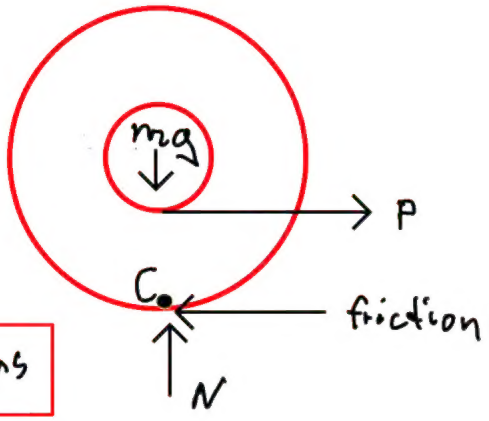
$\Sigma M_C = I_C \alpha$

$(P)(2\ meters) = (mk^2 + mR^2)\alpha$ ②

$\rightarrow$ **P = 260 Newtons**

$P - friction = 120$

$\rightarrow$ **friction = 140 $(-\hat{i})$ Newtons**



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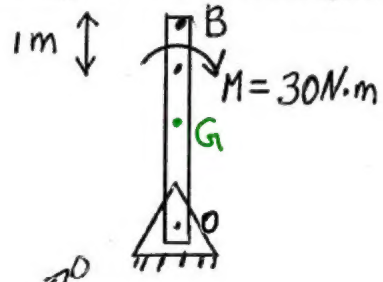
MIE 200F - Quiz number 7a - November 22/99
quiz duration = 25 minutes

A thin rod has a length of 5 meters and mass of 2 kg. It is pinned at one end, and is initially held stationary in the position shown in the diagram. A constant moment equal to 30 N·m is applied at the position shown. If the rod is released at the position shown in the diagram, what will be the speed of "B" when the rod reached a horizontal position?



$g = 9.81\ m/s^2 \downarrow$

$T_2 = T_1 + U_{1\to 2}$

$I_{c\ of\ g}$ for a rod $= m L^2/12$ $\qquad I_o = I_{c\ of\ g} + m d^2$

$I_{c\ of\ g}$ for a uniform disk $= m R^2/2$

$V_P = V_Q + V_{P/Q}$ $\qquad |V_{P/Q}| = \omega R_{PQ}$

$a_{P/Q} = -(\omega^2 R_{PQ})\hat{e}_r + (\alpha R_{PQ})\hat{e}_\theta$ $\qquad a_P = a_Q + a_{P/Q}$

$U_{gravity} = -mg\Delta h$ $\qquad U_{moment} = M\,\Delta\theta$

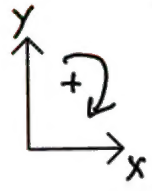
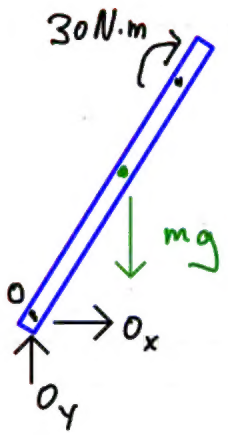
$T_2 = T_1 + U_{1\to 2}$

$\frac{1}{2} I_o \omega_2^2 = -mg\Delta h_G + M\Delta\theta$

$$I_o = \tfrac{1}{12} mL^2 + m\left(\tfrac{L}{2}\right)^2 = mL^2/3 = (2)(5)(5)/3 = 16.7$$

$$(\tfrac{1}{2})(16.7)\,\omega_2^2 = -(2)(9.81)(-2.5) + (30)(\pi/2)$$

$$8.33\,\omega_2^2 = 96.17 \Rightarrow \omega_2 = 3.4\,s^{-1}\ \circlearrowleft$$

$$\Rightarrow |\vec{V}_B| = (\omega_2)(L) = (3.4)(5) = 17\ m/s$$

Note: The location at which the moment M is applied does not matter.